Matematisk-fysiske Skrifter
udgivet af
Det Kongelige Danske Videnskabernes Selskab
Bind **1**, nr. 11

Mat. Fys. Skr. Dan. Vid. Selsk. **1**, no. 11 (1961)

EFFECT OF PAIRING CORRELATION ON ENERGIES AND β-TRANSITION PROBABILITIES IN DEFORMED NUCLEI

BY

V. G. SOLOVIEV

København 1961
i kommission hos Ejnar Munksgaard

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Selskabets sekretariat og postadresse: Dantes Plads 5, København V.

The address of the secretariate of the Academy is:

Det Kongelige Danske Videnskabernes Selskab,
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Selskabets kommissionær: EJNAR MUNKSGAARD's Forlag, Nørregade 6, København K.

The publications are sold by the agent of the Academy:

EJNAR MUNKSGAARD, *Publishers,*
6 Nörregade, Köbenhavn K, Denmark.

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CONTENTS

Synopsis

The effect of pairing correlations (or superfluidity) on the properties of strongly deformed nuclei is considered. It is taken into account that the pairing correlations depend on the nuclear state of excitation, and it is found that this "blocking effect" may essentially reduce the superfluidity in low lying states with quasi-particles present as compared with the ground state of even-even nuclei. In particular, the influence of pairing correlations on β-decay probabilities is computed and a comparison is made with available experimental data for odd-A nuclei.

PRINTED IN DENMARK

BIANCO LUNOS BOGTRYKKERI A/S

1. Introduction

The mathematical methods[1] developed in constructing the theory of superconductivity are very general. They permit one to take into account the residual fermion interactions leading to pairing correlations in a rather general form. Noting the similarity between the nuclear matter properties and the electronic structure of metals, N. N. Bogolubov[2] pointed out that nuclear matter can be superfluid. A. Bohr, B. Mottelson, and D. Pines[3] noticed that the nuclear excitation spectra and the spectra of superconducting states of metals are alike. They considered it reasonable to apply the methods employed in the theory of superconductivity to study the properties of finite nuclei.

The interactions between nucleons in a nucleus may be roughly separated into long range and short range parts. The long range part is responsible for the creation of the average nuclear field according to which the models of independent particles are constructed. The short range part leads mainly to the formation of the nucleon pairing correlations. In conjunction with the shell- and unified model the consideration of short range interactions between nucleons allows one to make progress in understanding the nuclear properties.

It is well known that the residual interactions between nucleons of a superconducting type cannot be included in the self-consistent potential.

This has been demonstrated in[4] where the self-consistent field is singled out in an explicit form. Note, that no modifications of the potential well can lead to the appearance of the effects which are responsible for the short range pairing interactions.

The investigations of the nucleon pairing correlations[5–9] provide an explanation of a number of nuclear properties which could not be accounted for in the framework of the model of independent particles. These investigations have shown that the residual short range forces between nucleons are attractive, and the ground state of any nucleus is the *superfluid* state in which pairing correlations are present. This state is energetically more favourable than that with successively filled levels of the average field in the model of independent particles. As is suggested in [10], the ground state of a light nucleus is the state with quadrupole nucleon correlations.

The superfluid properties of the ground- and excited nuclear states strongly affect a number of nuclear processes. These properties should be taken into account both in investigating the nuclear structure and in studying nuclear reactions.

This paper deals with the effect of pairing correlations on the properties of strongly deformed nuclei. We employ the method [11] by which the superfluid properties are calculated separately for each nuclear level. In section 2, the general equations of the model are given. In section 3, the calculations of the single-particle levels of odd–A nuclei are discussed, constants of the pairing interaction G are given, and basic assumptions for making detailed calculations are formulated. In section 4, the superfluid properties of a system containing only an even number of particles are investigated. In section 5, the superfluid corrections to the probabilities of beta transitions are calculated and are compared with the experimental data. In section 6, final conclusions are drawn regarding the comparison between the experimental data and the calculations made on the basis of the superfluid model of the nuclei.

2. Formulation of the Model

Using the average field of the independent-particle model, the superfluid nuclear model takes into account the short range part of nucleon-nucleon interactions in a nucleus leading to the pairing correlations under the following assumptions:

1. The residual interactions both between neutrons and between protons are described by the Hamiltonian of the form

$$H = \sum_{\zeta\sigma} \left\{ E_0(\zeta) - \lambda \right\} a^+_{\zeta\sigma} a_{\zeta\sigma} - G \sum_{\zeta,\zeta'} a^+_{\zeta+} a^+_{\zeta-} a_{\zeta'-} a_{\zeta'+}. \tag{1}$$

2. The main equations of the problem can be found with the aid of the variational principle proposed by Bogolubov [12], provided that the systems of equations characteristic of the superfluid properties of these states are obtained both for the ground- and excited nuclear states. The influence of the state of excitation on the pairing correlations is referred to as the blocking effect.

3. The mathematical method of solving the problem leads to the conservation of the number of particles, on the average,

$$n = \sum_{\zeta\sigma} \langle a^+_{\zeta\sigma} a_{\zeta\sigma} \rangle. \tag{2}$$

However, the calculations are made for quite definite nuclei.

The most important differences of the basic assumptions of the present model from those of the original method of investigating the pairing correlations [5]–[7] are

a) the difference between the superfluid nuclear properties in the ground- and excited states is taken into account,

b) the number of particles is conserved on the average for each nuclear state.

The blocking effect is most essential in the region of strongly deformed nuclei.

We first consider the equations for the characteristics of the superfluid state, as well as the wave functions and the energies of the ground- and excited states of the even and odd systems [5], [13].

Let us perform a canonical transformation

$$a_{\zeta\sigma} = U_\zeta \alpha_{\zeta -\sigma} + \sigma V_\zeta \alpha^+_{\zeta\sigma}, \tag{3}$$

with $U_\zeta^2 + V_\zeta^2 = 1$, and then find the mean value of the operator of the energy $\overline{H}$ by the state Ψ defined as $\alpha_{\zeta\sigma}\Psi = 0$:

$$\overline{H} = 2\sum_\zeta \{E_0(\zeta) - \lambda\} V_\zeta^2 - G\sum_{\zeta,\zeta'} U_\zeta V_\zeta U_{\zeta'} V_{\zeta'} - G\sum_\zeta V_\zeta^4. \tag{3^1}$$

Since the term $G\sum_\zeta V_\zeta^4$ makes a contribution to the self-consistent field, we carry out the renormalization

$$E(\zeta) = E_0(\zeta) - \frac{G}{2} V_\zeta^2 \tag{4}$$

and get

$$\overline{H} = 2\sum_\zeta \{E(\zeta) - \lambda\} V_\zeta^2 - G\sum_{\zeta,\zeta'} U_\zeta V_\zeta U_{\zeta'} V_{\zeta'}. \tag{5}$$

Let us determine U_ζ, V_ζ from the condition that $\overline{H}$ should be a minimum. As a result, we have

$$2\{E(\zeta) - \lambda\} U_\zeta V_\zeta - G(U_\zeta^2 - V_\zeta^2)\sum_{\zeta'} U_{\zeta'} V_{\zeta'} = 0. \tag{5^1}$$

We introduce a correlation function*)

$$C = G\sum_{\zeta'} U_\zeta V_\zeta \; (\equiv \Delta) \tag{6}$$

and determine

$$\left.\begin{aligned} U_\zeta^2 &= \frac{1}{2}\left\{1 + \frac{E(\zeta) - \lambda}{\varepsilon(\zeta)}\right\}, \qquad V_\zeta^2 = \frac{1}{2}\left\{1 - \frac{E(\zeta) - \lambda}{\varepsilon(\zeta)}\right\}, \\ \varepsilon(\zeta) &= \sqrt{C^2 + \{E(\zeta) - \lambda\}^2}. \end{aligned}\right\} \tag{6^1}$$

The wave function, the equations for determining C and λ, and the energy of the ground state of the even system are obtained as follows:

$$\Psi = \prod_\zeta (U_\zeta + V_\zeta a^+_{\zeta+} a^+_{\zeta-}) \Psi_0, \tag{7}$$

$$\frac{2}{G} = \sum_\zeta \frac{1}{\sqrt{C^2 + \{E(\zeta) - \lambda\}^2}}, \tag{8}$$

$$n = \sum_\zeta \left\{1 - \frac{E(\zeta) - \lambda}{\sqrt{C^2 + \{E(\zeta) - \lambda\}^2}}\right\}, \tag{9}$$

* The correlation function, here denoted by C, is in the literature often denoted by Δ (see, e. g., [6–9]).

$$\mathfrak{E} = \sum_{\zeta} E(\zeta)\left\{1 - \frac{E(\zeta)-\lambda}{\sqrt{C^2+\{E(\zeta)-\lambda\}^2}}\right\} - \frac{C^2}{G}, \tag{10}$$

where $a_{\zeta\sigma}\Psi_0 = 0$.

The wave functions, the energy, and the basic equations for the two-quasi-particle excited states of the even system are found to be

$$\Psi(f_1, f_2) = a^+_{f_1\sigma_1} a^+_{f_2\sigma_2} \prod_{\neq f_1, f_2} (U_\zeta(f_1, f_2) + V_\zeta(f_1, f_2)\, a^+_{\zeta+} a^+_{\zeta-})\Psi_0, \qquad f_1 \neq f_2 \tag{11}$$

$$\Psi(f, f) = (U_f(f, f)\, a^+_{f+} a^+_{f-} - V_f(f, f)) \cdot \prod_{\zeta \neq f} (U_\zeta(f, f) + V_\zeta(f, f)\, a^+_{\zeta+} a^+_{\zeta-})\Psi_0, \tag{11^1}$$

$$\left.\begin{aligned} \mathfrak{E}(f_1, f_2) &= E(f_1) + E(f_2) + \frac{G}{2}(V_{f_1}(f_1, f_2)^2 + V_{f_2}(f_1, f_2)^2) \\ &+ \sum_{\zeta \neq f_1, f_2} 2\, E(\zeta)\, V_\zeta(f_1, f_2)^2 - \frac{C(f_1, f_2)^2}{G}, \end{aligned}\right\} \tag{12}$$

$$\frac{2}{G} = \sum_{\zeta \neq f_1, f_2} \frac{1}{\sqrt{C(f_1, f_2)^2 + \{E(\zeta) - \lambda(f_1, f_2)\}^2}}, \tag{13}$$

$$n = 2 + \sum_{\zeta \neq f_1, f_2} \left\{1 - \frac{E(\zeta) - \lambda(f_1, f_2)}{\sqrt{C(f_1, f_2)^2 + \{E(\zeta) - \lambda(f_1, f_2)\}^2}}\right\}. \tag{14}$$

The ground state of the system consisting of the odd number of particles is the state with one quasi-particle on the K-level which is the last filled single-particle level of the average field at $G = 0$. As excited states of the odd system, consider both the single-quasi-particle and three-quasi-particle states. For the single-quasi-particle ground- and excited states we get the wave function

$$\Psi(f) = a^+_{f\sigma} \prod_{\zeta \neq f} (U_\zeta(f) + V_\zeta(f)\, a^+_{\zeta+} a^+_{\zeta-})\Psi_0, \tag{15}$$

the energy

$$\mathfrak{E}(f) = E(f) + \frac{G}{2} V_f(f)^2 + \sum_{\zeta \neq f} 2\, E(\zeta)\, V_\zeta(f)^2 - \frac{C(f)^2}{G}, \tag{16}$$

and the basic equations

$$\frac{2}{G} = \sum_{\zeta \neq f} \frac{1}{\sqrt{C(f)^2 + \{E(\zeta) - \lambda(f)\}^2}} \tag{17}$$

$$n = 1 + \sum_{\zeta \neq f} \left\{1 - \frac{E(\zeta) - \lambda(f)}{\sqrt{C(f)^2 + \{E(\zeta) - \lambda(f)\}^2}}\right\}. \tag{18}$$

The wave function, the energy, and the basic equations of the three-quasi-particle excited states of the odd system are found to be

$$\Psi(f_1, f_2, f_3) = a^+_{f_3\sigma_3} a^+_{f_2\sigma_2} a^+_{f_1\sigma_1} \cdot \prod_{\zeta \neq f_1, f_2, f_3} (U_\zeta(f_1, f_2, f_3) + V_\zeta(f_1 f_2 f_3)\, a^+_{\zeta+} a^+_{\zeta-})\Psi_0, \tag{19}$$

provided

$$\left.\begin{aligned}\mathfrak{E}(f_1, f_2, f_3) = E(f_1) + E(f_2) + E(f_3) + \frac{G}{2}[V_{f_1}(f_1, f_2, f_3)^2 + \\ V_{f_2}(f_1, f_2, f_3)^2 + V_{f_3}(f_1, f_2, f_3)^2] + \sum_{\zeta \neq f_1, f_2, f_3} 2E(\zeta)V_\zeta(f_1, f_2, f_3)^2 - \frac{C(f_1, f_2, f_3)^2}{G},\end{aligned}\right\} \tag{20}$$

$$\frac{2}{G} = \sum_{\zeta \neq f_1, f_2, f_3} \frac{1}{\sqrt{C(f_1, f_2, f_3)^2 + \{E(\zeta) - \lambda(f_1, f_2, f_3)\}^2}}, \tag{21}$$

$$n = 3 + \sum_{\zeta \neq f_1, f_2, f_3} \left\{1 - \frac{E(\zeta) - \lambda(f_1, f_2, f_3)}{\sqrt{C(f_1, f_2, f_3)^2 + \{E(\zeta) - \lambda(f_1, f_2, f_3)\}^2}}\right\}. \tag{22}$$

In order to determine the main superfluid characteristics of the above-mentioned states, i.e. the correlation functions C and the chemical potentials λ, it is necessary, as in [13, 15], to solve the corresponding systems of equations by means of an electronic computer. To investigate the general properties of the superfluid states of nuclei, we take as levels of the average field the energy levels of Nilsson's scheme [14].

3. Pairing Energies and Single-Particle Levels of the Odd-A Nuclei

The single-particle level spectra of the majority of the odd, and a number of even-even, strongly-deformed nuclei as well as the superfluid corrections to β- and γ-transitions, both in the rare-earth and transuranic regions, have been calculated according to the superfluid model of the nuclei in [11, 13, 15]. Slightly corrected Nilsson schemes [16], close to that presented in [9], have been used as single-particle levels of the average field, with the exception of the $11/2-[505]$ neutron state, which was lowered by approximately $0.3\, \hbar \mathring{\omega}_0$ ($\hbar \mathring{\omega}_0 = 41\, A^{-1/3}$ MeV).

As regards the influence of the superfluidity on the behaviour of the single-particle levels of the odd-A nucleus, it was shown that

1) The superfluidity does not alter, as a rule, the spin of the ground state presented by Nilsson's scheme.

2) The excitation energy of the system decreases rather quickly with G and the compression of the single-particle levels does not occur uniformly.

3) Hole and particle excited states behave differently with increasing G. However, the sequence of hole (particle) levels with respect to each other remains unchanged.

The energy difference between the excited f and the ground f_0 state of the system with an odd number of particles is written as

$$\left.\begin{aligned}\mathfrak{E}(f) - \mathfrak{E}(f_0) = \varepsilon_f(f) - \varepsilon_{f_0}(f_0) + \frac{C(f)^2 - C(f_0)^2}{2G} \\ + \frac{G}{2}(V_f(f)^2 - V_{f_0}(f_0)^2) - \sum_\zeta \{\varepsilon_\zeta(f) - \varepsilon_\zeta(f_0)\}.\end{aligned}\right\} \tag{23}$$

One can easily see that, if $C(f) = C(f_0)$ and $\lambda(f) = \lambda(f_0)$, then, neglecting a small addition due to the renormalization of the average field, we get

$$\mathfrak{E}(f_0) - \mathfrak{E}(f_0) = \varepsilon_f(f_0) - \varepsilon_{f_0}(f_0). \tag{23^1}$$

This result has been obtained in the original formulation of the pairing correlations [5–7], where the superfluid characteristics of the ground- and excited states were considered to be identical. In this formulation, however, the average number of particles varies by two as one goes from a level with $E(s) \ll \lambda$ to one with $E(s) \gg \lambda$.

The pairing interaction constants of the neutron G_n and proton G_z systems are found from the experimental values for the pairing energies by the formula

$$P_N = 2\,\mathfrak{E}(Z, N-1) - \mathfrak{E}(Z, N) - \mathfrak{E}(Z, N-2) \tag{24}$$

or by the more strict formula

$$P_N = \frac{1}{2}\left\{3\,\mathfrak{E}(Z, N-1) + \mathfrak{E}(Z, N+1) - 3\,\mathfrak{E}(Z, N) - 3\,\mathfrak{E}(Z, N-2)\right\}, \tag{24^1}$$

where the corresponding experimental data are available. In a previous analysis, the following expectation values of the pairing interaction constants were obtained from a comparison of the calculated values of the pairing energies with the experimental data: rare-earth region [15] $G_N = 0.024\,\hbar\,\mathring{\omega}_0$, $G_Z = 0.026\,\hbar\,\mathring{\omega}_0$, transuranic region [13] $G_N = 0.020\,\hbar\,\mathring{\omega}_0$, $G_Z = 0.022\,\hbar\,\mathring{\omega}_0$, the number of the summed levels being 24–28 in Eqs. (8), (9) and (17), (18).

With these parameters, the single-particle levels of many odd nuclei were calculated in [13, 15] and found to be in better agreement with the experimental data than those given in the Nilsson scheme. However, in view of the strong dependence of the calculated levels of the odd-A nuclei on the behaviour of the average field levels, the main emphasis was placed upon the investigation of the single-particle level density. It has been shown in [13, 15] that, both in the rare-earth and in the transuranic regions, the density of the low-energy single-particle levels agrees with the experimental data and is approximately two times greater than that of Nilsson's scheme, which is consistent with the analysis carried out in [17]. Note, that the effect of increasing the level density is associated with the superfluid properties of the ground- and excited states. It cannot be obtained by changing the behaviour of the single-particle levels in the independent model.

Due to the uncertainties in the details of the average field, we shall in the present treatment use the experimental data on the levels of odd-A nuclei, together with the empirical data on pairing energies, to determine the single-particle level spectrum in the average field. With this aim in view, we correct the behaviour of some levels in Nilsson's scheme near the K-level and choose the pairing interaction constant G in order to obtain the single-particle spectra of odd nuclei and pairing energies, which would agree with experimental data. The precise position of levels far from the K-state has only little effect on the superfluid properties of the system. Therefore, as long as the behaviour of the levels near the K-state is taken according to the experimental data, the calculations in which the wave functions are not used do not depend on a concrete choice of the potential of the average field. Thus, the calculations of the relative magnitudes depend only very weakly on the parameters of the average field. The main drawback and limitation on the accuracy of the calculations by the suggested scheme is that the change of the average field for neighbouring nuclei is not taken into account, since one and the same assembly of the single-particle levels has to be used in calculating the superfluid characteristics of a number of nuclei. However, in such an approach, there appears a possibility of investigating the change of the average field in passing from one nucleus to another, as well as the role of other factors which were not taken into account and, above all, the interactions of quasi-particles.

It is to be noted that for the given system of the average field levels and for the fixed magnitude of the pairing interaction constants the calculations based on the superfluid model are *entirely unambiguous.*

Let us investigate the properties of the strongly deformed nuclei in the region $156 \leqslant A \leqslant 188$. We divide the nuclei under consideration into two groups: the first group $156 \leqslant A \leqslant 174$ $(63 \leqslant Z \leqslant 70)$; $92 \leqslant N \leqslant 104)$, the second group $174 \leqslant A \leqslant 188$ $(70 \leqslant Z \leqslant 76,\ 104 \leqslant N \leqslant 112)$. In each group we choose one set of the single-particle levels of the average field and one pairing interaction constant both for the neutron and proton systems, in order to make the single-particle spectra of odd-A nuclei and pairing energies agree with the experimental data. In Table 1 we write down the relative configurations of the most important levels for the first and the second groups of nuclei. Note that the behaviour of some levels of the first group is different from that

TABLE 1
Single-particle energy levels.

Neutron system				Proton system			
N	Assigned orbital	Energy ($\hbar\mathring{\omega}_0$) I	Energy ($\hbar\mathring{\omega}_0$) II	Z	Assigned orbital	Energy ($\hbar\mathring{\omega}_0$) I	Energy ($\hbar\mathring{\omega}_0$) II
93.......	3/2− [521]	0	0	63.....	5/2+ [413]	0	0
95.......	5/2+ [642]	0.04	0.04	65.....	3/2+ [411]	0.04	0.04
97.......	5/2− [523]	0.07	0.07	67.....	7/2− [523]	0.12	0.12
99.......	7/2+ [633]	0.22	0.22	69.....	1/2+ [411]	0.20	0.20
101.......	1/2− [521]	0.24	0.24	71.....	9/2− [514]	0.42	0.30
103.......	5/2− [512]	0.29	0.29	73.....	7/2+ [404]	0.31	0.31
105.......	7/2− [514]	0.41	0.33	75.....	5/2+ [402]	0.36	0.43
107.......	9/2+ [624]	0.48	0.41	77.....	1/2+ [400]	0.52	0.52
109.......	1/2− [510]		0.48				
111.......	3/2− [512]		0.55				
113.......	7/2− [503]		0.59				

of the corresponding levels of the second group, which is not unreasonable because of different values for the magnitudes of deformation.

Since the experimental values of the pairing energies change from nucleus to nucleus, we make a rather rough averaging. After the corresponding calculations we get the following values of the pairing interaction constants:

$$\left.\begin{aligned} &\text{The first group} \quad 156 \leqslant A \leqslant 174 \\ &G_N = 0.021\,\hbar\mathring{\omega}_0 \approx 0.16 \text{ MeV}, \\ &G_Z = 0.023\,\hbar\mathring{\omega}_0 \approx 0.17 \text{ MeV}, \end{aligned}\right\} \quad (25)$$

$$\left.\begin{aligned} &\text{the second group} \quad 174 \leqslant A \leqslant 188 \\ &G_N = 0.020\,\hbar\mathring{\omega}_0 \approx 0.145 \text{ MeV}, \\ &G_Z = 0.021\,\hbar\mathring{\omega}_0 \approx 0.152 \text{ MeV}. \end{aligned}\right\} \quad (25^1)$$

Thus, over the whole region $156 \leqslant A \leqslant 188$ one can consider

$$\left.\begin{aligned} G_N &= \frac{26}{A} \text{ MeV}, \\ G_Z &= \frac{28}{A} \text{ MeV}. \end{aligned}\right\} \quad (26)$$

The summation in the equations for determining C and λ is being performed over 36 single-particle levels*. Note that the values of the pairing interaction constants depend on the number of the levels in the summation. The physical properties of the system are practically independent of the number of the levels summed up when their number exceeds 10–12 levels both higher and lower than the K-state. By increasing the number of the summed levels the values of the constants G_n and G_z became smaller by $(0.03$—$0.04)\,\hbar\mathring{\omega}_0$ compared to the values in [15], where the summation was made over 24 levels. However, in spite of decreasing the constants G_n and G_z, the magnitudes of the correlation functions C changed insignificantly compared to their values in [15], while the ratios $C(K)/C$ somewhat increased.

The choice of parameters (26) is consistent with that of Nilsson and Prior [9], considering the larger number of levels included in the summations by these authors.

The magnitudes of the correlation functions C of the ground state of the even proton systems in the region $63 \leqslant Z \leqslant 76$ are monotonically falling with Z from $C = 0.13\,\hbar\mathring{\omega}_0$ down to $C = 0.11\,\hbar\mathring{\omega}_0$. As for the ratio $C(K)/C$, it changes irregularly within the interval 0.50—0.75. For the neutron system in the range $92 \leqslant N \leqslant 112$ the magnitudes of the correlation functions C of the ground states with the even number of particles change within the limit $(0.11$—$0.14)\,\hbar\mathring{\omega}_0$, and the ratios $C(K)/C$ assume the values within the interval 0.6—0.8.

A choice of two sets of the average field levels and of two values of G for a large

* I wish to thank Dr. S. Nilsson who pointed out that it is reasonable to increase the number of the summed levels.

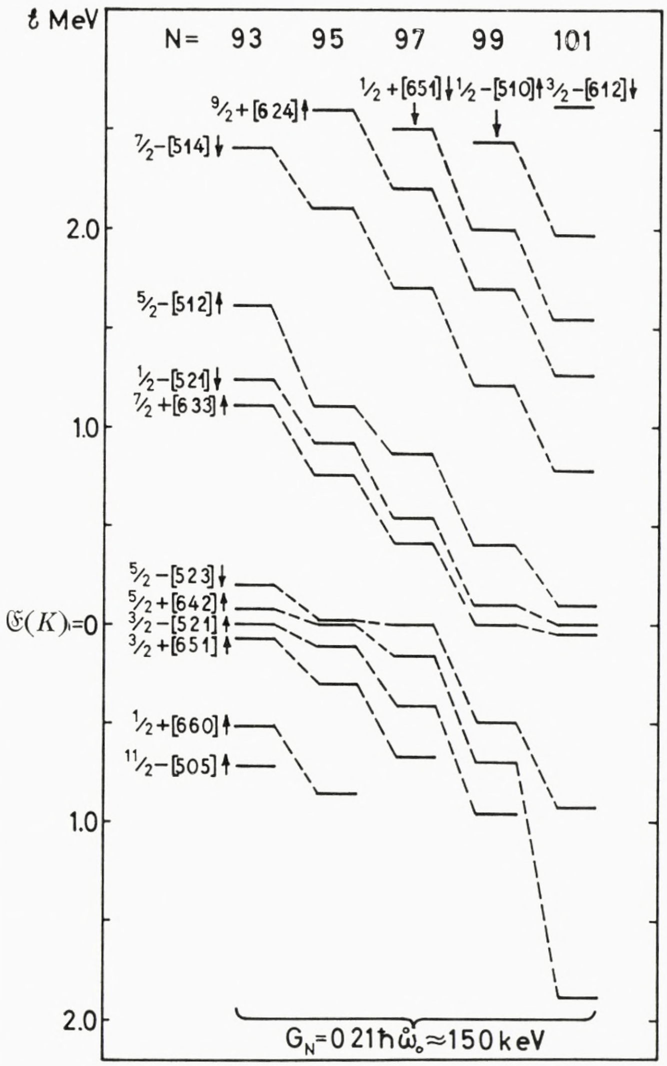

Fig. 1.

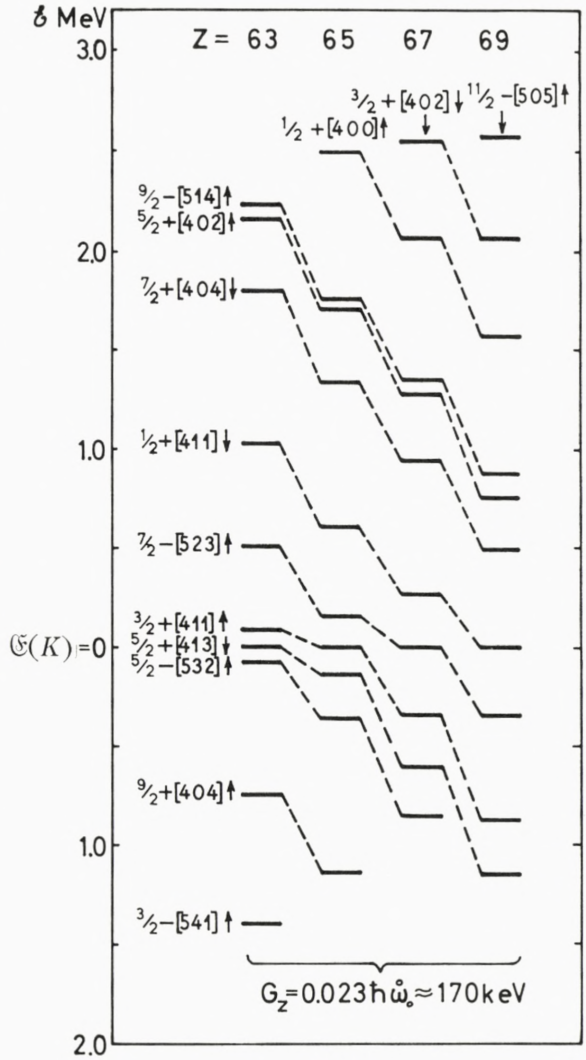

Fig. 2.

group of nuclei is a rather rough approximation, as the behaviour of the average field levels, the equilibrium deformation, and the pairing energies change noticeably. We do so to exclude, first, the arbitrariness in calculating the even-even nuclei spectra, second, to be able to compare the relative values of log ft for the β-decays and to find log ft in the even nuclei according to the experimental data on the β-transitions in the odd nuclei. We act also in such a way because the single-particle levels of odd nuclei are known to be relatively few, and the available experimental data did not give evidence for the necessity of forming more than two groups.

The calculated single-particle spectra of the odd-A nuclei, illustrated in Figs. 1 and 2, show particle excited states above $\mathfrak{E}(K) = 0$ and hole states below $\mathfrak{E}(K) = 0$.

The calculations lead to a fairly acceptable description of the behaviour of the single-particle levels of the odd-A nuclei. The exception is the change in the sequence of the levels $7/2+[404]$ and $9/2-[514]$ in some odd-Z nuclei and the change in the spin of the ground state of the odd-N nuclei when $N = 95$. When $N < 93$ the scheme we accepted fails to give correct values for the spins of the ground states of some odd-N nuclei. Therefore, for the case with $N = 91$, we substituted the level $11/2-[505]$ in place of the level $5/2-[523]$, and moved this level, together with $5/2+[642]$ and $3/2-[521]$, down each by one step, thus retaining the energy spectrum.

Among the excited states of the system consisting of the odd number of particles, there must also be observed the three-quasi-particle states besides the single-quasi-particle ones. The energy of the three-quasi-particle states has to be, as a rule, somewhat higher than the excitation energy corresponding to the even system. However, in the particular cases when the particle and hole levels of the average field come very close to the K-level, i.e., in the vicinity of the crossing point of the three-single-particle levels, the energy of the three-quasi-particle state $(K-I, K, K+I)$ becomes noticeably lower. Consider, for example, the $K-1$, K and $K+1$ levels of Dy^{161}, which are $3/2-[521]$, $5/2+[642]$ and $5/2-[523]$, respectively. The energy difference between the $(K+1)$ and $(K-1)$ levels is found experimentally to be 0.1 MeV. Assuming that the pairing correlations are absent in the three-quasi-particle states $(K-1, K, K+1)$, we find that these excited states must have spins and parities $3/2+$, $7/2+$, and $13/2+$, each with an energy about 1 MeV. Some of these levels may be observable by the Coulomb excitation of Dy^{161} in contrast to the situation in β-decay, where the probabilities of these levels being populated is sufficiently large only in the transition from those excited states in which two-quasi-particles are situated on any two of the three states $K-1$, K, and $K+1$.

4. Superfluid Properties of Systems Consisting of an Even Number of Particles

The advantage of the superfluid model over the independent-particle one is exhibited especially clearly by the properties of the even-even nuclei. The single-particle level spectra calculated in [11, 13, 15] reflect the basic regularities in the behaviour of the even-even nuclear levels, In this section, we consider the general properties of the superfluid nuclear model, viz., the dependence of the superfluid state characteristics, and the behaviour of the even system levels on the magnitude of the pairing interaction constant G, the superfluid properties of the two-quasi-particle excited states of the system, and the specific features of the $0+$ states, etc.

As an example, we take the neutron system with $N = 106$, e.g., Hf^{178}, and investigate the behaviour of the ground- and two-quasi-particle excited states as the pairing interaction constant G increases. As mentioned above, the experimental value of G is found to be $0.020\,\hbar\mathring{\omega}_0$. For $G = 0.012\,\hbar\mathring{\omega}_0$, the correlation function C is very small, while the energy of the ground state is approximately equal to its value with $G = 0$. Thus, for $G = 0.012\,\hbar\overset{\circ}{\omega}_0$, there are practically no pairing correlations. For $G = 0.016\,\hbar\overset{\circ}{\omega}_0$ we have $C = 0.058\,\hbar\overset{\circ}{\omega}_0 = 0.42$ MeV, and the energy is decreased by $0.017\,\hbar\overset{\circ}{\omega}_0 = 0.12$ MeV compared with the case when $G = 0$, although the negative potential energy changes to a greater extent, viz., $C^2/G = 1.5$ MeV. Fig. 3 shows the values of the ground-state energy and the magnitudes of the gap $2\,C$ for G equal to

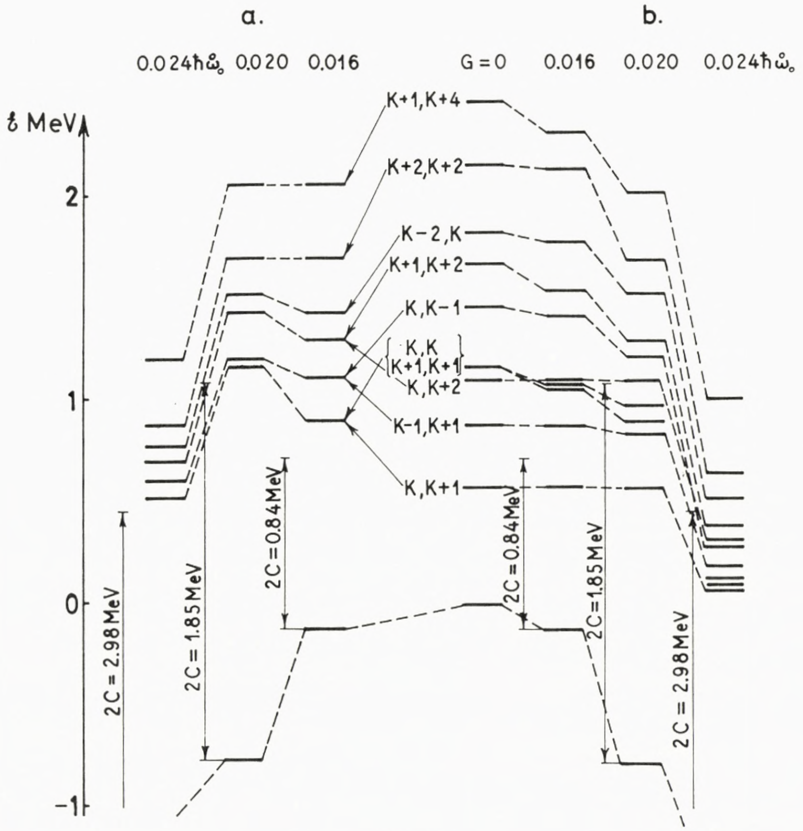

Fig. 3.

0.016; 0.020; $0.024\,\hbar\overset{\circ}{\omega}_0$. For $G = 0.028\,\hbar\overset{\circ}{\omega}_0$, the energy of the ground state decreases by $0.70\,\hbar\overset{\circ}{\omega}_0 = 5.1$ MeV, and $C = 0.28\,\hbar\overset{\circ}{\omega}_0 = 2.1$ MeV. For $G = 0.032\,\hbar\overset{\circ}{\omega}_0$, the correlation function of the ground state $C = 0.37\,\hbar\overset{\circ}{\omega}_0 = 2.7$ MeV, and the energy decreases by $1.17\,\hbar\overset{\circ}{\omega}_0 = 8.5$ MeV, C^2/G being equal to 30.5 MeV.

Consider the behaviour of the two-quasi-particle excited states of the system depending on G. With this aim in mind we represent in Fig. 3 case b) the energies of the excited states calculated according to the superfluid nuclear model with the aid of an electronic computer. In Fig. 3 case a) we give the energies of the excited states calculated by the formula

$$\varepsilon(K_1) + \varepsilon(K_2)$$

according to the original formulation of the pairing correlations [5, 6]. We designate by $(K,\ K+2)$ the state of the system with one quasi-particle on the K-level, and the second one on the $K+2$ level, where K is the last filled single-particle level of the average field at $G = 0$. Note that in the present model the number of particles is conserved on the average, and all the calculated excited states may be referred to the same system of particles.

As is seen from Fig. 3, the behaviour of the two-quasi-particle excited state energies as a function of G, calculated by the superfluid nuclear model, is very different for small G from their behaviour in the case a), where an increase in the energy of some lower states is observed with G up to $G = 0.020\ \hbar\mathring{\omega}_0$, which is due to the incorrectness of the mathematical methods used. In the case b), the energies of both the ground- and excited states are decreasing monotonically with G, although the degree of their energy decrease is different. This is connected with the circumstance that, in the present model, the specific superfluid properties of the individual quasi-particle levels is taken into account.

Let us analyse the behaviour of the correlation functions $C\,(K_1,\ K_2)$ of the two-particle excited states. To this end, we write down, in Table 2, the changes in the ratios $C\,(K_1\ K_2)/C$ with increasing G. In the range of the values for G from 0.016 up to 0.024, there is a great difference in the magnitude of the correlation function of the excited states, in some low-excited states the pairing correlations being absent even for $G = 0.016\ \hbar\mathring{\omega}_0$. For $G = 0.028\ \hbar\mathring{\omega}_0$ and larger, they are little different from each other and less than the magnitude of the correlation function C of the ground state by approximately (15—20) %. Thus, the difference between the correlation functions of the excited states which is taken into account by the superfluid nuclear model is essential at $G = 0.020\ \hbar\mathring{\omega}_0$, which corresponds to the actual nuclear forces. This is so because, for $G = 0.020\ \hbar\mathring{\omega}_0$ in the interval $2\,C$, there are 4—5 levels of the average field which are most effective. The superfluid properties of the excited states depend strongly on which two-single-particle levels the quasi-particles are. If one or two quasi-particles are in the levels far from K, then the effect of their elimination in (13), (14) decreases. It is seen from Table 2 that, for $G = 0.016\ \hbar\mathring{\omega}_0$ where $2\,C = 0.84$ MeV, the $K+3$, $K+4$ levels are not very effective, and the ratios

$$C\,(K+2,\,K+3)/C \qquad C\,(K+1,\,K+4)/C$$

and others are large. With increasing G, the number of the most effective levels increases, and for $G = 0.020\ \hbar\mathring{\omega}_0$ the ratio $C\,(K+2,\,K+3)/C$ and others decreases. The number of the most effective levels becomes larger with G. Therefore, the ratios $C\,(K_1,\,K_2)/C$ are getting greater. For $G = 0.028\ \hbar\mathring{\omega}_0$ and larger, the number of the most effective levels is great and the elimination of any two of them in (13) and (14) leads, practically, to the same results.

To show the importance of the blocking effect we give ratios $C\,(K_1,\,K_2)/C$ in Tables 3 and 4 for the same neutron and proton systems at a value of G, which cor-

TABLE 2

	G in units of ($\hbar \mathring{\omega}_0$)				
	0.016	0.020	0.024	0.028	0.032
$C\ (K, K+1)/C$	0	0.23	0.67	0.74	0.83
$C\ (K-1, K+1)/C$	0	0.46	0.70	0.78	0.84
$C\ (K, K+2)/C$	0	0.53	0.71	0.79	0.84
$C\ (K, K)/C$	0.56	0.62	0.71	0.79	0.84
$C\ (K-1, K)/C$	0.62	0.67	0.74	0.79	0.84
$C\ (K+1, K+2)/C$	0.76	0.67	0.73	0.79	0.84
$C\ (K-2, K-1)/C$	0.72	0.75	0.78	0.81	0.85
$C\ (K+1, K+4)/C$	0.80	0.72	0.76	0.81	0.85
$C\ (K+2, K+3)/C$	0.87	0.76	0.78	0.82	0.85

TABLE 3

	$G_n = 0.021\ \hbar \mathring{\omega}_0$					$G_n = 0.020\ \hbar \mathring{\omega}_0$	
	$N = 92$	$N = 94$	$N = 96$	$N = 98$	$N = 100$	$N = 106$	$N = 108$
$C\ (K, K+1)/C$	0.43	0.47	0	0	0	0.23	0.24
$C\ (K-1, K+1)/C$	0.61	0.54	0.16	0.06	0.56	0.46	0.58
$C\ (K, K+2)/C$	0.55	0.54	0.64	0	0.10	0.53	0.50
$C\ (K, K)/C$	0.53	0.53	0.28	0.71	0.37	0.62	0.62
$C\ (K-1, K)/C$	0.66	0.58	0.40	0.72	0.59	0.67	0.71
$C\ (K+1, K+2)/C$	0.63	0.61	0.69	0.77	0.36	0.67	0.65
$C\ (K-2, K-1)/C$	0.75	0.68	0.54	0.76	0.75	0.75	0.81
$C\ (K+1, K+4)/C$	0.73	0.73	0.71	0.84	0.54	0.72	0.68
$C\ (K+2, K+3)/C$	0.71	0.75	0.84	0.82	0.62	0.76	0.73
$C\ (\hbar \mathring{\omega}_0)$	0.131	0.129	0.121	0.109	0.111	0.127	0.131

TABLE 4

	$G_z = 0.023\ \hbar \mathring{\omega}_0$			$G_z = 0.021\ \hbar \mathring{\omega}_0$	
	$Z = 64$	$Z = 66$	$Z = 68$	$Z = 72$	$Z = 74$
$C\ (K, K+1)/C$	0.20	0	0	0.006	0.005
$C\ (K-1, K+1)/C$	0.43	0.25	0	0.39	0.006
$C\ (K, K+3)/C$	0.54	0.50	0.39	0.47	0.39
$C\ (K, K)/C$	0.48	0.54	0.53	0.19	0.66
$C\ (K-1, K)/C$	0.55	0.57	0.60	0.48	0.67
$C\ (K+1, K+2)/C$	0.60	0.67	0.68	0.54	0.69
$C\ (K-2, K-1)/C$	0.68	0.65	0.71	0.69	0.75
$C\ (K+1, K+4)/C$	0.68	0.73	0.70	0.61	0.72
$C\ (K+2, K+3)/C$	0.73	0.79	0.79	0.74	0.78
$C\ (\hbar \mathring{\omega}_0)$	0.131	0.127	0.121	0.114	0.112

Table 5

I state	II state	$\underset{\zeta \neq f_1 f_2}{\Pi}(U_\zeta(f_1 f_1)\, U_\zeta(f_2 f_2) + V_\zeta(f_1 f_1)\, V_\zeta(f_2 f_2))^2$					
		$G = 0.012$	0.016	0.020	0.024	0.028	0.032 $\hbar\omega_0$
$\vert 0>$	$K-1, K-1$	0.0001	0.58	0.82	0.90	0.93	0.96
$\vert 0>$	$K+1, K+1$	0.08	0.72	0.82	0.86	0.92	0.95
K, K	$K+2, K+2$	0.007	0.28	0.64	0.88	0.96	0.98
$K+1, K+1$	$K-1, K-1$	0.09	0.44	0.71	0.90	0.97	0.99
$K+2, K+2$	$K-1, K-1$	0.001	0.20	0.59	0.84	0.94	0.97
$K+2, K+2$	$K+1, K+1$	0.99	0.99	0.98	0.98	0.99	1.00

responds to the actual nuclear forces. As is seen from these tables, the influence of quasi-particles on the superfluid properties of the system is very essential.

In investigating the effect of the ground and excited state superfluidity on the β- and γ-transition probabilities [13, 20], we see that in the expressions for the superfluid corrections the factors appear of the form (see (31) below)

$$\underset{\zeta \neq f_1 f_2}{\Pi}(U_\zeta U_\zeta' + V_\zeta V_\zeta')^2, \tag{27}$$

which are not unity due to the difference in the superfluid properties of the initial and final states. Let us decide whether these factors are important. Therefore, in Table 5, we give the changes in G which are products of the form

$$\underset{\zeta \neq f_1 f_2}{\Pi}(U_\zeta (f_1 f_1)\, U_\zeta (f_2 f_2) + V_\zeta (f_1 f_1)\, V_\zeta (f_2 f_2))^2, \tag{27^1}$$

where the initial and final states are the $0+$ states. In Table 5, the ground state of the system is denoted by $|0>$ while the two-quasi-particle excited state with both quasi-particles on the K$-$1 level is denoted by $K-1$, $K-1$, etc. As we can see from this table, the expression (27^1) increases with G, and for $G = 0.032\ \hbar\mathring{\omega}_0$ it is approaching unity. We give in Table 6 some values of (27) for G_Z and G_N which correspond to the real nuclear forces. As is seen from Tables 5 and 6, values of (27) change within the interval 0.5—1.0, though most of its values are within the range 0.7—0.9. Thus, in investigating the β- and γ-transitions in the strongly deformed nuclei, it is necessary to calculate the products of form (27) for each case.

One of the most important results of the present calculations is the falling of the energy of one, and in some cases, of several excited states lower than the magnitude of the gap 2 C. It is shown in [13] that, in the excited state of the (K, $K+1$) even system (i. e. in the state where one quasi-particle is situated on the K-level, and the other on the next higher $K+1$ level), the superfluidity decreases considerably. This is connected with the fact that the correlated pairs according to the Pauli principle cannot occupy the K and $K+1$ levels. Therefore, in the states which the pairs can populate, there appears a large gap for the strongly deformed nuclei. If the number of the states

Table 6

Neutron system $G_N = \frac{26}{A}$ MeV				
even system		odd system		$\underset{\zeta \neq f_1 f_2}{\Pi} (U_\zeta U'_\zeta + V_\zeta V'_\zeta)^2$
N	state	N'	state	
92	$K+1, K+2$	93	K'	0.84
94	$K-1, K$	95	$K'-1$	0.98
96	$K, K+1$	95	K'	0.56
98	$K, K+2$	97	K'	0.67
98	$K, K+1$	99	K'	0.72
98	$K+1, K+3$	99	K'	0.70
98	ground	99	$K'-1$	0.83
			K'	0.86
			$K'+1$	0.91
			$K'+3$	0.99

Proton system $G_Z = \frac{28}{A}$ MeV				
even system		odd system		$\underset{\zeta \neq f_1 f_2}{\Pi} (U_\zeta U'_\zeta + V_\zeta V'_\zeta)^2$
Z	state	Z'	state	
			$K'-1$	0.95
			K'	0.91
64	ground	63		
			$K'+1$	0.89
			$K'+2$	0.82
64	$K, K+3$	63	K'	0.99
64	$K, K+1$	65	K'	0.76
66	$K, K+1$	65	K'	0.58
66	$K-1, K+1$	67	K'	0.88
66	$K+1, K+2$	67	K'	0.71
68	$K, K+2$	67	K'	0.95
68	$K+1, K+2$	69	K'	0.60

below the gap is equal to the number of particles, it is energetically unfavourable for the pairs to populate the $K+2$ and higher levels, and the superfluidity in the $(K, K+1)$ state then becomes considerably less. The calculated values of the energy of the $(K, K+1)$ state for a number of nuclei are noticeably smaller than the magnitude of $2\,C$ and agree well with the corresponding experimental data, as is seen from Table 7, where we give the value of the gap $2\,C$, the calculated energy levels $(K, K+1)$, and the experimental data which are analyzed in [24]. The agreement of the theory with the experiment as far as the depression of the $(K, K+1)$ state energy below the gap is concerned gives evidence for the importance of the blocking effect, provided we can neglect effects associated with the interaction between quasi-particles. In

TABLE 7
Energy of state $(K, K+1)$

Nuclei	System	$K\pi$	Gap $2C$ (MeV)	Energy (MeV)	
				calculated	observed
W^{184}	proton	2−	1.61	1.3	1.150
W^{182}	proton	2−	1.61	1.3	1.290
W^{182}	neutron	4−	1.89	1.5	1.554
Hf^{180}	proton	8−	1.66	1.0	1.142
Hf^{178}	proton	8−	1.66	1.0	1.148
Hf^{178}	neutron	8−	1.85	1.5	1.480
Yb^{172}	proton	3+	1.80	1.4	1.664
Yb^{172}	neutron	3+	1.65	1.3	1.174
		$2+(\Sigma=1)$			1.468
Er^{168}	proton	$3-(\Sigma=1)$	1.82	1.3	1.543
Er^{168}	neutron	$3-(\Sigma=1)$	1.64	1.1	1.095
Dy^{162}	neutron	5−	1.83	1.3	1.485
Dy^{160}	proton	2−	1.90	1.4	1.260
Dy^{160}	neutron	1−	1.96	1.5	(1.20)
Gd^{156}	proton	4+	2.0	1.45	1.511
Gd^{156}	neutron	1−	2.0	1.5	1.240

some cases, such interactions may give rise to collective effects, especially for the $2+$ levels which may acquire the character of γ-vibrations, with an associated additional depression of the energy.

Among the excited states of the even system, the $0+$ states with two-quasi-particles on the same single-particle level of the average field occupy a special place. The wave functions of these states are non-orthogonal with respect to each other and to the wave function of the ground state of the system, viz.:

$$\left.\begin{aligned} \langle f_1, f_1 \mid f_2, f_2 \rangle = {} & \pm (U_{f_1}(f_1, f_1)\, V_{f_1}(f_2, f_2) - U_{f_1}(f_2, f_2)\, V_{f_1}(f_1, f_1)) \\ & \times (U_{f_2}(f_1, f_1)\, V_{f_2}(f_2, f_2) - U_{f_2}(f_2, f_2)\, V_{f_2}(f_1, f_1)) \\ & \times \prod_{\zeta \neq f_1, f_2} (U_\zeta(f_1, f_1)\, U_\zeta(f_2, f_2) + V_\zeta(f_1, f_1)\, V_\zeta(f_2, f_2)). \end{aligned}\right\} \quad (28)$$

The wave functions of the remaining two-particle excited states are orthogonal both with respect to each other and to the wave function of the ground state. So, the mathematical difficulties connected with the conservation of the number of particles on the average are concentrated, so to say, on the $0+$ states among which one state is superfluous. Let us analyse the dependence on G (Table 8) of the magnitudes of the non-orthogonal wave functions of the ground- and excited states. When the pairing correlations are absent, i.e., when $G = 0$, the number of $0+$ states must decrease by one in comparison with the number of states calculated by the superfluid nuclear model. It is seen from Table 8 that, for $G \to 0$, the states (K, K) and $(K+1, K+1)$

TABLE 8
Non-orthogonality of the 0+ states

	G ($\hbar\overset{\circ}{\omega}_0$)					
	0.012	0.016	0.020	0.024	0.028	0.032
$< O \mid K-1, K-1 >$	10^{-5}	0.13	0.18	0.12	0.07	0.03
$< O \mid K, K >$	10^{-5}	0.27	0.24	0.11	0.05	0.02
$< O \mid K+1, K+1 >$	0.02	0.31	0.24	0.09	0.05	0.01
$< O \mid K+2, K+2 >$	0.02	0.09	0.16	0.12	0.07	0.06
$< K, K \mid K+1, K+1 >$	0.92	0.52	0.20	0.02	0.003	0.001
$< K, K \mid K+2, K+2 >$	0.004	0.07	0.07	0.02	0.005	0.002
$< K+1, K+1 \mid K-1, K-1 >$	0.02	0.10	0.07	0.02	0.004	0.002

coincide, and the remaining ones become mutually orthogonal. With increasing G the magnitude of the non-orthogonal wave functions of the states (K, K) and $(K+1, K+1)$ decreases, but the non-orthogonality of the wave functions of other low-excited 0+ states rises. For large values of G, the non-orthogonality of the wave functions of the low-excited states becomes smaller since the number of the mutually non-orthogonal 0+ states increases. It is seen from Fig. 3 that, for small G, the energies of the (K, K) and $(K+1, K+1)$ states practically coincide; with increasing G there appears a slight splitting. When the values of G correspond to $G = 0.020\, \hbar\overset{\circ}{\omega}_0$, one can roughly consider that there is one 0+ state with an averaged excitation energy instead of the two (K, K) and $(K+1, K+1)$ states.

For the case $N = 106$, $G = 0.020\, \hbar\, \omega_0$ considered above, we investigate the dependence on $E(\zeta)$ of the function $V_\zeta^2 = \frac{1}{2}\left\{1 - \frac{E(\zeta) - \lambda}{\sqrt{C^2 + \{E(\zeta) - \lambda\}^2}}\right\}$, the expression $[C^2 + \{E(\zeta) - \lambda\}^2]^{-1/2}$ under the sum in (8), and the function $\frac{C^2}{C^2 + \{E(\zeta) - \lambda\}^2}$ entering the expression for the average quadratic fluctuation of the number of particles

$$(\Delta n)^2 = \sum_\zeta{}' \frac{C^2}{C^2 + \{E(\zeta) - \lambda\}^2}. \tag{29}$$

Fig. 4 (p. 20) shows the dependence of the value of these functions on the energy, which is measured from that of the K-level in the units of $\hbar\overset{\circ}{\omega}_0$, MeV, and in the magnitudes of $C = 0.127\, \hbar\overset{\circ}{\omega}_0 = 0.925$ MeV. The vertical lines indicate the position of the average field levels. The values of V_ζ^2 and $C^2 \cdot [C^2 + \{E(\zeta) - \lambda\}^2]^{-1}$ should be read off from the left-hand axis of the ordinate, and the values $[C^2 + \{E(\zeta) - \lambda\}^2]^{-1/2}$ along the right-hand one. We see from Fig. 4 that, at the energies higher than about $3\,C$ or $4\,C$, measured from the K-level, the magnitudes of the considered functions are monotonically approaching the corresponding limit. The region of the average field levels which is most important for the pairing correlation effect is within the interval $6\,C$–$8\,C$ near the K state and comprises 15–20 levels. However, in (8), (9) and other equations,

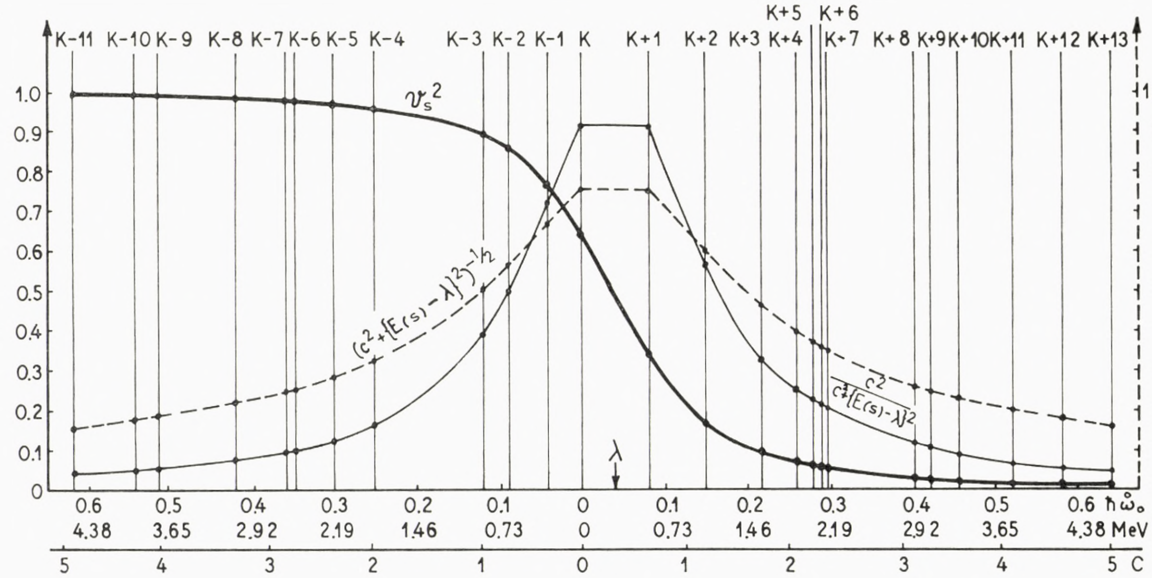

Fig. 4.

the summation region should be increased up to $10C$ and even more. If the correlation function of the ground state is weakly dependent on the number of the levels over which the summation is being made, then the ratio $C(K)/C$ of the correlation function of the ground state of the odd system $C(K)$ to C is more sensitive to the cut-off since [9]

$$\frac{C(K)}{C} \approx 1 - \frac{1}{\sum_{\zeta} \frac{C^3}{\varepsilon_\zeta^3}}. \tag{30}$$

If, in making the summation, we restrict ourselves to the region less than $8\,C$, then the ratio $C(K)/C$ becomes too small. For instance, in the calculations for the proton system with $Z = 93$, when a sufficient number of levels was taken higher than K, then for a lower cut-off at $3.5\,C$ below K, we get $C(K)/C = 0.5$ for $C = 0.09\,\hbar\,\mathring{\omega}_0$, whereas for a cut-off at $7\,C$ below K, one obtains $C(K)/C = 0.8$ for $C = 0.08\,\hbar\,\mathring{\omega}_0$. By further increasing the number of the summed levels the ratio changes very weakly. In the present calculations the summation is taken over 36 levels of the average field.

The change in the superfluid properties of the system in the transition from the ground- to excited states of the even system will undoubtedly affect the magnitudes of the moments of inertia of the ground- and excited states calculated according to the superfluid model of a nucleus. The moment of inertia for the ground state depends upon the superfluid properties both of the ground- and excited states, i. e., upon the superfluid characteristics of the whole system. The moment of inertia of the system in an excited state is dependent on the superfluid properties both of the given and other states. Therefore, a sharp decrease in the magnitude of the correlation function

$C(K_1, K_2)$ for the given excited state, e.g., for the $(K, K+1)$ state, will not necessarily lead to the same considerable change in the magnitude of the moment of inertia.

The superfluid properties of the strongly deformed nuclei depend very much upon the magnitude of the pairing interaction constant G. If G were twice as little as the value corresponding to the nuclear forces in heavy nuclei, then the pairing correlations would practically be absent. If, on the other hand, G were twice as much, many features of the nuclei would considerably alter, and the shell structure would be, at least, strongly masked. The differences in the superfluid properties of the two-quasi-particle excited states relative to each other and to the ground state of the system are essential in the region of strongly deformed nuclei and are outside the errors of the method.

Thus, the specific features of the superfluid model of a nucleus are important for the values of G which correspond to the residual nuclear forces, and when the behaviour of the single-particle levels of the average field is like that in the strongly deformed nuclei.

5. Superfluid Corrections and Additional Classification of β-transition Probabilities

It is shown in [11, 13] that the role of the superfluid corrections to the β-and γ-transition probabilities in strongly deformed nuclei may be appreciable. In this section we formulate general rules of constructing the corrections to β-transitions due to the superfluidity of the ground- and excited states. Besides keeping Alaga's selection rules classification of the probabilities for β-decay of strongly-deformed nuclei, we introduce an additional selection rule. The role of the superfluid corrections is investigated by analyzing $\log ft$ for the β-transitions between identical pairs of the single-particle states in different nuclei.

The matrix element describing the β-decay of a complex nucleus is written symbolically as

$$\left.\begin{aligned} M \sim \Psi^*_{2n_N} \Psi^*_{2n_Z+1}(\zeta_2) \sum_{\nu, \nu'} \langle \nu \,|\, \Gamma \,|\, \nu^1 \rangle a^+_\nu b_{\nu'} \\ \times \Psi_{2n_Z} \Psi_{2n_N+1}(\zeta_1) = \langle \zeta_2 \,|\, \Gamma \,|\, \zeta_1 \rangle R^{1/2}. \end{aligned}\right\} \tag{31}$$

Here, $\langle \zeta_2 | \Gamma | \zeta_1 \rangle$ is the single-particle matrix element of the transition, and $R^{1/2} = (\Psi^*_{2n_N} \Psi_{2n_N})(\Psi^*_{2n_Z} \Psi_{2n_Z})$, where Ψ_n is the wave function of the N-particle system. The values ft characterizing the β-decay are obtained in the form

$$ft = \frac{\text{Const}}{|\langle \zeta_2 \,|\, \Gamma \,|\, \zeta_1 \rangle|^2} R^{-1}, \tag{32}$$

R being represented as $R = R_z R_n$. The quantities R_z and R_n describe the change in the proton and neutron configurations of the nucleus associated with the β-transition in the form

$$R_i = \gamma \mathop{\Pi}_{\zeta \neq f_1, \cdots f_n} (U_\zeta U'_\zeta + V_\zeta V'_\zeta)^2 \tag{33}$$

with the functions U_ζ, V_ζ referring to the initial and U'_s, V'_s to the final states. The product in (33) extends over the levels in which there are no quasi-particles in initial as well as final states. In the special case of two quasi-particles in the same level the corresponding expression is given below (34).

The more alike the superfluid properties of the initial and final states, the closer the product (33) approaches unity. In the formulation [5, 6] of the pairing correlations, this product is equal to unity. Further, if the number of paired particles in the initial and final states is the same, as, e.g., in the β-decay ${}^{108+1}_{72}\mathrm{Hf}^{181} \to {}^{108}_{72+1}\mathrm{Ta}^{181}$, then $\gamma = U_f^2$. If the number of paired nucleons varies in the course of the decay, as in ${}^{110}_{72+1}\mathrm{Ta}^{183} \to {}^{108+1}_{74}\mathrm{W}^{181}$, then $\gamma = V_f^2$, f being referred to the level on which a quasi-particle either disappeared or appeared. The functions U_f^2 or V_f^2 in (33) characterize the superfluid properties of the system with a smaller number of quasi-particles. So, for instance, in the β-decay of the odd system into the ground state of the even one, V_f^2 and U_f^2 are referred to the even system, while in the β-decay of the single-particle odd state into the two-quasi-particle excited state, V_f^2 and U_f^2 are referred to the odd system, etc.

Consider the case when the pairing interaction constant G tends to zero, i.e., when the superfluid model passes into the independent-particle model. Then the correction R_i takes one of the two values $R_i = 1$ or $R_i = 0$, provided $R_i = 1$; this corresponds to the case when the β-decay occurs without any change in the configuration of all the nucleons except one, whereas in the case $R_i = 0$ the β-decay is proceeding accompanied by a change in the configuration of more than one nucleon in the independent-particle model. For the β-decay in which the number of pairs remains unaltered, $R_i = 1$ for the particle transitions, and $R_i = 0$ for the hole ones, while for the β-decay in which the number of pairs changes by unity, $R_i = 1$ for the hole transitions and $R_i = 0$ for the particle ones. The particle transitions are called the transitions in which a quasi-particle either disappears or appears on the single-particle levels f whose energy is higher than λ referred to the system with the smaller number of quasi-particles. For the hole transitions, the energies of the single-particle levels f are lower than λ.

Let us make [20] an additional (in comparison with the selection rules formulated in [16]) classification of the β-decay probabilities of the strongly deformed complex nuclei, viz., we divide all the β-transitions into three groups:

$$\begin{array}{lll} \text{group I} & R_i(G=0) = 1, & 0 < R_i(G \neq 0) < 1 \\ \text{group II} & R_i(G=0) = 0, & 0 < R_i(G \neq 0) < 1 \\ \text{group III} & R_i(G=0) = 0, & R_i(G \neq 0) = 0. \end{array}$$

The first group includes

a) those β-decays whose initial and final states are the ground states of the system,

Table 9

Allowed unhindered beta transitions of the odd nuclei

Nuclei	Class.	R	log $(ft)_e$	log $(ft)_r$	log $[(ft)_e R\eta]$
$\zeta_z = \{ 7/2-[523] \} \rightleftarrows \zeta_n = \{ 5/2-[523] \}$					
$^{100}_{67}Ho^{167} \rightarrow {}^{99}_{68}Er^{167}$	I, I	0.52	4.8[16]	**4.8**	4.4
$^{98}_{69}Tm^{167} \leftarrow {}^{97}_{70}Yb^{167}$	I, I	0.58	$\lesssim 5$ [16]	4.7	$\lesssim 4.7$
$^{96}_{65}Tb^{161} \leftarrow {}^{97}_{64}Gd^{161}$	I, I	0.26	$\lesssim 4.9$[16, 25]	5.0	~ 4.3
$^{92}_{67}Ho^{159} \rightarrow {}^{93}_{66}Dy^{159}$	I, I	0.39	$\lesssim 5$ [16]	4.9	$\lesssim 4.5$
$^{94}_{67}Ho^{161} \rightarrow {}^{95}_{66}Dy^{161}$	I, I	0.31	$\lesssim 4.5$[16]	5.0	4.1
$^{96}_{67}Ho^{163} \leftarrow {}^{95}_{68}Er^{163}$	I, I	0.36	$\lesssim 5$ [16]	4.8	$\lesssim 4.6$
$^{98}_{67}Ho^{165} \leftarrow {}^{97}_{68}Er^{165}$	I, I	0.42	$\lesssim 5$ [16]	4.8	$\lesssim 4.6$
$\zeta_z = \{ 9/2-[514] \} \rightleftarrows \zeta_n = \} 7/2-[514] \}$					
$^{104}_{71}Lu^{175} \leftarrow {}^{105}_{70}Yb^{175}$	I, I	0.32	4.7[25]	**4.7**	4.2
$^{106}_{73}Ta^{179} \leftarrow {}^{105}_{74}W^{179}$	I, I	0.44	4.6[25]	4.6	4.2

b) the particle transitions when the number of pairs remains unaltered,

c) the hole transitions when the number of pairs changes by unity.

For example, the β-transitions [18] into the state 7/2−[503] Hf^{177} with an energy of 1060.1 keV, into the states 9/2 − [505] and 7/2 − [503] Hf^{175} with energies of 1226.7 keV and 1045.5 keV, respectively.

The second group includes

a) the hole transitions when the number of particle pairs does not change,

b) the particle transitions when the number of particle pairs changes by unity. For the β-decays referred to the second group, the superfluid model allows non-zero transition probabilities, while these transitions are strictly forbidden in the independent-particle model. It is worth while noting that the corrections R_i calculated by the superfluid nuclear model, which are referred to the first and second groups and which are associated with the β-transitions to the low-excited nuclear states ($\lesssim$ 0.3MeV), are equal to each other within an order of magnitude; in the transitions to the strongly excited states (1 MeV and higher) they differ very much.

The analysis of the experimental data shows that there are more than 20 already established β-transitions referred to the second group (see Tables 9–11). The observation of these β-transitions with intensities of the expected order of magnitude provides further evidence for the presence of short-range pairing interactions.

While the first and the second groups incorporate those β-decays in which only one quasi-particle in the proton (neutron) systems disappears or appears and the configuration of the remaining particles is left unaltered, the third group includes

a) the transitions in which the number of quasi-particles of the proton (neutron) system changes by more than unity;

Table 10
Allowed hindered beta transitions of odd nuclei

Nuclei	Class.	R	$\log (ft)_e$	$\log (ft)_r$	$\log [(ft)_e R\eta]$
$\zeta_z = \{ 5/2-[532] \} \rightleftarrows \zeta_n = \{ 3/2-[521] \}$					
${}^{94}_{65}Tb^{159} \leftarrow {}^{95}_{64}Gd^{159}$	II, I	0.07	6.7[16]	**6.7**	5.6
${}^{94}_{67}Ho^{161} \leftarrow {}^{93}_{68}Er^{161}$	I, I	0.53	$\sim$ 6 [23]	5.8	5.7
$\zeta_z = \{ 5/2+[413] \} \rightleftarrows \zeta_n = \{ 5/2+[642] \}$					
${}^{92}_{63}Eu^{155} \rightarrow {}^{91}_{64}Gd^{155}$	I, II	0.20	7.3[16]	–	6.6
$\zeta_z = \{ 3/2+[411] \} \rightleftarrows \zeta_n = \{ 5/2+[642] \}$					
${}^{96}_{65}Tb^{161} \rightarrow {}^{95}_{66}Dy^{161}$	I, I	0.40	6.8[25]	–	6.4
$\zeta_z = \{ 7/2-[523] \} \rightleftarrows \zeta_n = \{ 5/2-[512] \}$					
${}^{102}_{69}Tm^{171} \leftarrow {}^{103}_{68}Er^{171}$	II, I	0.15	6.3[15]	–	5.5
$\zeta_z = \{ 7/2+[404] \} \rightleftarrows \zeta_n = \{ 7/2+[633] \}$					
${}^{102}_{73}Ta^{175} \rightarrow {}^{103}_{12}Hf^{175}$	I, II	0.12	6.7[18]	**6.7**	5.8
${}^{104}_{73}Ta^{177} \rightarrow {}^{105}Hf^{177}$	I, II	0.06	7.8[18]	7.0	6.6
$\zeta_z = \{ 7/2+[404] \} \rightleftarrows \zeta_n = \{ 9/7+[624] \}$					
${}^{108}_{73}Ta^{181} \leftarrow {}^{107}_{74}W^{181}$	I, I	0.34	6.6[25]	**6.6**	6.2
${}^{106}_{71}Lu^{177} \rightarrow {}^{107}_{70}Yb^{177}$	I, I	0.41	6.2[16]	6.5	5.9
${}^{106}_{71}Lu^{177} \rightarrow {}^{105}_{72}Hf^{177}$	I, I	0.15	6.3[16]	7.0	5.5
${}^{104}_{73}Ta^{177} \rightarrow {}^{105}_{72}Hf^{177}$	I, I	0.33	7.9[18]	6.7	7.4
${}^{106}_{73}Ta^{179} \rightarrow {}^{107}_{72}Hf^{179}$	I, I	0.25	6.9[16]	6.8	6.3
${}^{108}_{75}Re^{183} \leftarrow {}^{107}_{76}Os^{183}$	I, I	0.46	7.1[22]	6.5	6.9

b) the transitions in which, besides the change in the number of quasi-particles by unity, the configuration of other quasi-particles changes.

The superfluid nuclear model is a model of independent quasi-particles. Therefore, the transitions associated with the changes of configuration of the quasi-particles are strictly forbidden. It would be of interest to have experimental data on the degree of forbiddenness of the transitions referred to the third group. To this end, the probability should be found for β-decay of the single-quasi-particle state of the odd system into such a two-quasi-particle excited state of the even system so that all three quasi-particles would be on different single-particle levels of the average field.

For the β-decay of the two-quasi-particle state of the even system, where the quasi-particles are on the same level f, to the state of the odd system with a quasi-particle on the level ν the superfluid correction is found to be

$$\left.\begin{aligned} R_i = {} & \delta_{f\nu}\, U_f\ (f,f)^2 \prod_{\zeta \neq f} (U_\zeta(ff)\, U_\zeta(f) + V_\zeta(ff)\, V_\zeta(f))^2 \\ & + (1-\delta_{\nu f})\, V_\nu(f,f)^2\, (U_f(ff)\, V_f(\nu) - V_f(ff)\, U_f(\nu))^2 \\ & \times \prod_{\zeta \neq f,\nu} (U_\zeta(ff)\, U_\zeta(\nu) + V_\zeta(ff)\, V_\zeta(\nu))^2, \end{aligned}\right\} \quad (34)$$

when the number of paired nucleons remains unaltered. The first component in (34) differs from the corrections considered above in so far as the factor $U_f(f,f)$ is referred to the state with the greater number of quasi-particles. The second component in (34) gives the non-zero contribution to the β-transitions referred to the third group. Its appearance in (34) is likely to be due to the above-considered non-orthogonality of the $0+$ states.

The same corrections must be included in the cross sections of several nuclear reactions, e.g., the cross section of the stripping reactions (dp) is proportional to R_N. If the final states of the nuclei consist of an odd number of neutrons, then the correction has the form

$$U_f^2 \prod_{\zeta \neq f} (U_\zeta\, U_\zeta(f) + V_\zeta\, V_\zeta(f))^2 \tag{35}$$

and particle excitations of the final state nuclei are more probable than hole excitations.

However, if the final states of the nuclei consist of an even number of neutrons, then the correction to the cross section for formation of the ground state is different from that for the excited states. For the ground state we have the factor

$$V_K^2 \prod_{\zeta \neq K} (U_\zeta\, U_\zeta(K) + V_\zeta\, V_\zeta(K))^2, \tag{36}$$

while for the excited two quasi-particle states we have

$$U_f(K)^2 \prod_{\zeta \neq K,f} (U_\zeta(K)\, U_\zeta(K,f) + V_\zeta(K)\, V_\zeta(K,f))^2. \tag{36^1}$$

Let us calculate the superfluid corrections to β-transitions between single-particle states of the odd nuclei. The calculations of R_i, formula (33), are made with the use of an electronic computer and take into account the scheme of single-particle levels and the values of G_N and G_Z given in section 3. The result of the calculations are shown in Figs. 5 and 6. Neutron corrections R_N are shown in Fig. 5, case a) corresponding to transitions in which the number of pairs remains unaltered $(2n \rightleftarrows 2n+1)$, case b) corresponding to β-transitions in which the number of pairs changes by one $(2n \rightleftarrows 2n-1)$. We distinguish between the different curves by stating the number of neutrons of the odd system, and we denote the position of the ground state on any one such curve by a double circle ◉. The axis of abscissa shows the number of single-particle states whose quantum numbers are given in Table 1. The proton corrections are similarly shown in Fig. 6.

The aim is, on the one hand, to clear up the role of the superfluid corrections, and, on the other, to account for the change in the experimental values of $\log ft$ in

TABLE 11
First forbidden unhindered transitions of odd nuclei

Nuclei	Class.	R	$\log(ft)_e$	$\log(ft)_r$	$\log[(ft)_e R\eta]$
$\zeta_z = \{3/2+[411]\} \rightleftarrows \zeta_n = \{3/2-[521]\}$					
${}^{94}_{65}\mathrm{Tb}^{159} \leftarrow {}^{95}_{64}\mathrm{Gd}^{159}$	I, I	0.24	6.7[16]	**6.7**	6.1
${}^{96}_{65}\mathrm{Tb}^{161} \rightarrow {}^{95}_{66}\mathrm{Dy}^{161}$	I, I	0.47	6.8[16]	6.4	6.5
$\zeta_z = \{7/2-[523]\} \rightleftarrows \zeta_n = \{7/2+[633]\}$					
${}^{98}_{67}\mathrm{Ho}^{165} \leftarrow {}^{99}_{66}\mathrm{Dy}^{165}$	I, I	0.33	6.2[16]	**6.2**	5.7
${}^{100}_{67}\mathrm{Ho}^{167} \rightarrow {}^{99}_{68}\mathrm{Er}^{167}$	I, I	0.23	6.0[16]	6.3	5.4
$\zeta_z = \{1/2+[411]\} \rightleftarrows \zeta_n = \{3/2-[521]\}$					
${}^{94}_{67}\mathrm{Ho}^{161} \leftarrow {}^{93}_{68}\mathrm{Er}^{161}$	II, I	0.18	6.4[23]	–	6.0
$\zeta_z = \{1/2+[411]\} \rightleftarrows \zeta_n = \{1/2-[510]\}$					
${}^{100}_{69}\mathrm{Tm}^{169} \leftarrow {}^{101}_{68}\mathrm{Er}^{169}$	I, I	0.26	6.4[25]	**6.4**	5.8
${}^{102}_{69}\mathrm{Tm}^{171} \rightarrow {}^{101}_{70}\mathrm{Yb}^{171}$	I, I	0.30	6.3[25]	6.3	5.8
${}^{58}_{69}\mathrm{Tm}^{167} \rightarrow {}^{99}_{68}\mathrm{Er}^{167}$	I, I	0.36	6.5[25]	6.3	6.1
$\zeta_z = \{1/2+[411]\} \rightleftarrows \zeta_n = \{1/2-[510]\}$					
${}^{108}_{75}\mathrm{Re}^{183} \leftarrow {}^{107}_{76}\mathrm{Os}^{183}$	I, II	0.27	6.3[22]	**6.3**	5.7
${}^{108}_{73}\mathrm{Ta}^{181} \leftarrow {}^{109}_{72}\mathrm{Hf}^{181}$	II, I	0.07	7.1[16]	6.9	6.0
$\zeta_z = \{1/2+[411]\} \rightleftarrows \zeta_n = \{3/2-[512]\}$					
${}^{112}_{75}\mathrm{Re}^{187} \leftarrow {}^{113}_{74}\mathrm{W}^{187}$	II, I	0.02	7.7[21]	–	6.0
$\zeta_z = \{9/2-[514]\} \rightleftarrows \zeta_n = \{9/2+[624]\}$					
${}^{108}_{75}\mathrm{Re}^{183} \leftarrow {}^{107}_{76}\mathrm{Os}^{183}$	I, I	0.46	6.2[22]	**6.2**	5.9
${}^{108}_{73}\mathrm{Ta}^{181} \leftarrow {}^{107}_{74}\mathrm{W}^{181}$	I, I	0.37	6.8[25]	6.3	6.4
$\zeta_z = \{7/2+[404]\} \rightleftarrows \zeta_n = \{7/2-[514]\}$					
${}^{102}_{73}\mathrm{Ta}^{175} \rightarrow {}^{103}_{72}\mathrm{Hf}^{175}$	I, I	0.31	6.4[18]	**6.4**	5.9
${}^{104}_{71}\mathrm{Lu}^{175} \leftarrow {}^{105}_{70}\mathrm{Yb}^{175}$	I, I	0.32	6.4[16]	6.4	5.9
${}^{105}_{71}\mathrm{Lu}^{177} \rightarrow {}^{105}_{72}\mathrm{Hf}^{177}$	I, I	0.27	6.6[25]	6.5	6.0
${}^{104}_{73}\mathrm{Ta}^{177} \rightarrow {}^{105}_{72}\mathrm{Hf}^{177}$	I, I	0.21	6.6[18]	6.6	5.9
$\zeta_z = \{7/2+[404]\} \rightleftarrows \zeta_n = \{7/2-[503]\}$					
${}^{102}_{73}\mathrm{Ta}^{175} \rightarrow {}^{103}_{72}\mathrm{Hf}^{175}$	I, I	0.41	6.2[18]	**6.2**	5.8
${}^{104}_{73}\mathrm{Ta}^{177} \rightarrow {}^{105}_{172}\mathrm{Hf}^{177}$	I, I	0.40	6.4[18]	6.2	6.0
${}^{110}_{73}\mathrm{Ta}^{183} \rightarrow {}^{109}_{74}\mathrm{W}^{183}$	I, II	0.11	6.9[16]	6.8	5.9
${}^{112}_{73}\mathrm{Ta}^{185} \rightarrow {}^{111}_{74}\mathrm{W}^{185}$	I, II	0.19	6.5[19]	6.5	5.8

(continued)

TABLE 11 (continued)

Nuclei	Class.	R	$\log(ft)_e$	$\log(ft)_r$	$\log[(ft)_e R\eta]$
$\zeta_z = \{7/2+[404]\} \rightleftarrows \zeta_n = \{9/2-[505]\}$					
${}^{102}_{73}Ta^{175} \rightarrow {}^{103}_{72}Hf^{175}$	I, I	0.41	6.4[18]	**6.4**	6.0
${}^{112}_{73}Ta^{185} \rightarrow {}^{111}_{74}W^{185}$	I, II	0.16	7.3[19]	6.8	6.5
$\zeta_z = \{5/2+[402]\} \rightleftarrows \zeta_n = \{3/2-[512]\}$					
${}^{112}_{75}Re^{187} \leftarrow {}^{113}_{74}W^{187}$	I, I	0.30	7.9[21]	**7.9**	7.4
${}^{110}_{75}Re^{185} \leftarrow {}^{111}_{74}W^{185}$	I, I	0.38	7.5[16]	7.8	7.1
${}^{108}_{75}Re^{183} \rightarrow {}^{109}_{74}W^{183}$	I, I	0.46	7.4[25]	7.5	7.1

passing between the identical single-particle states in different nuclei and to calculate $\log ft$ for the β-transitions in the even nuclei by using the experimental data on the β-transitions in the odd nuclei. To eliminate the influence of the average field as much as possible we analyse the β-transitions between the pairs of identical single-particle states in different nuclei. In such an approach, of course, the influence of the single-particle matrix element $\langle f_1 | \Gamma | f_2 \rangle$ on the relative values of $\log ft$ is not entirely excluded, since in the transition from one nucleus to another the average field changes slightly.

Following the additional classification of the β-transitions introduced above, we denote the transitions referred to the first group by I, those referred to the second group by II, and those referred to the third group by F. According to the generally accepted classification of the β-transitions we shall write down the additional one, first, for the proton system, and then for the neutron one. For example, *a h I II* implies that the proton transition is assigned to the first group, and the neutron one to the second group.

Consider the β-transitions in the odd-A nuclei. In Tables 9, 10, and 11 the experimental data and the results of the calculations are shown. To facilitate the comparison of the β-transition probabilities between the pairs of identical single-particle states in different nuclei, we have put all the relevant quantities together. In the first column of Tables 9–11 we have written down the parent and daughter nuclei. The additional classification is introduced in the second column. In the third column the superfluid correction R is given and, in the fourth, the experimental values of $\log(ft)_e$ with references, are shown. In column 5 are given the calculated values of $\log(ft)_r$ relative to the first value of the given set which has been normalized to the experimental value. In the calculated value has been included the statistical factor $\eta = \langle I_i K_i \lambda_\nu | I_f K_f \rangle^2$, where (λ, ν) is the multipole order of the transition. Note that, if the average field does not alter in passing from one nucleus to another, then the change of $\log(ft)_e$ should be accounted for by the superfluid corrections.

Since the asymptotic selection rules are based on the independent-particle model in the form of Nilsson's potential, it is convenient for this classification to consider

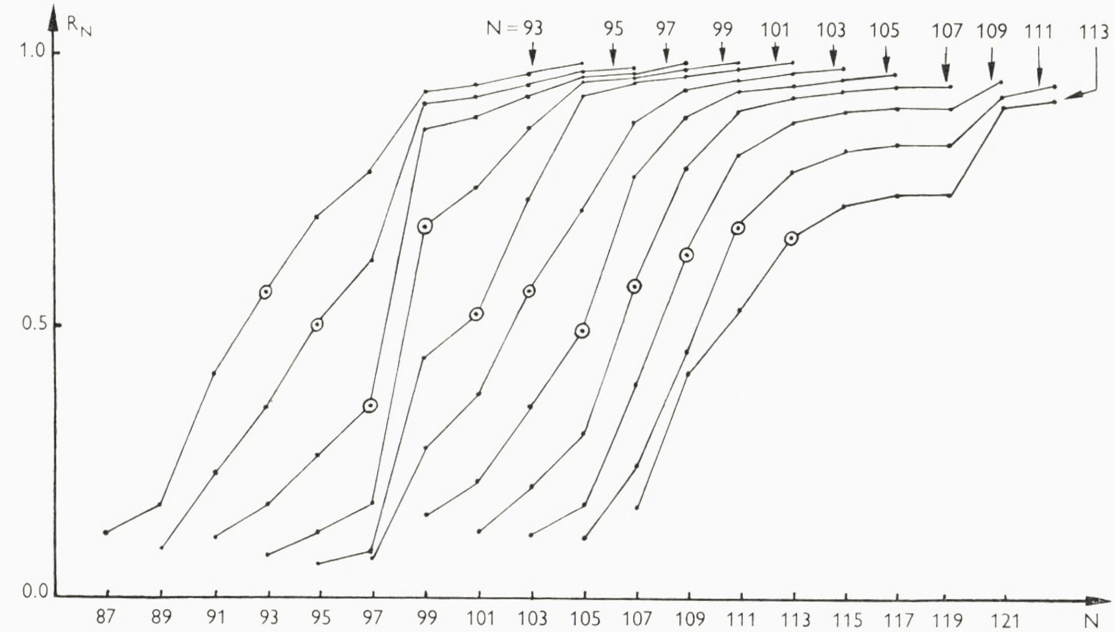

Fig. 5 a.

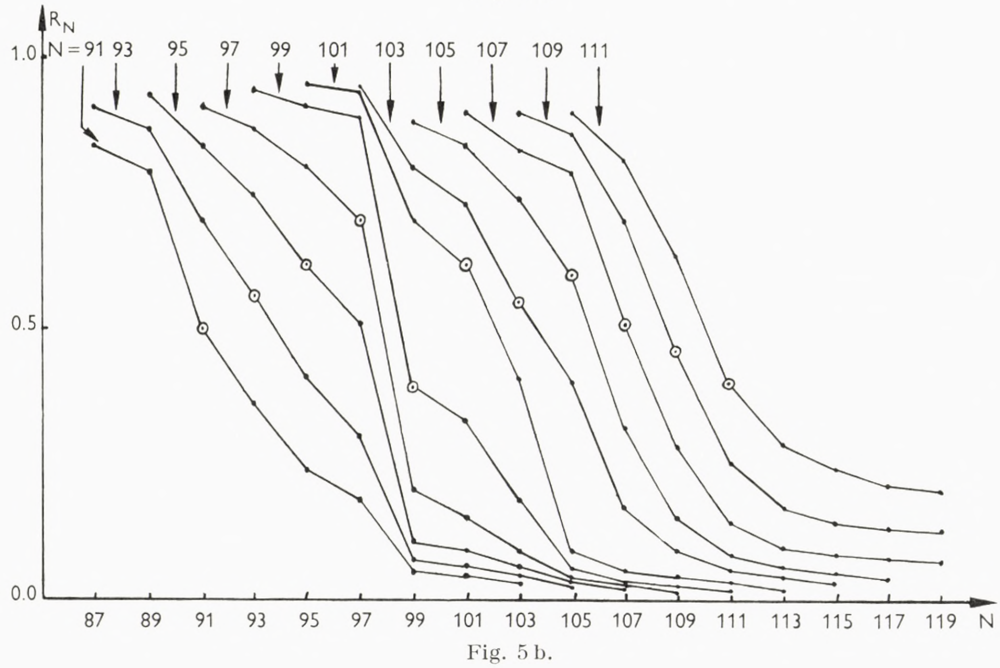

Fig. 5 b.

$\log[(ft)_e R\eta]$ rather than $\log(ft)_e$, thereby excluding the effect of superfluidity and the statistical factor η. The values of $\log[(ft)_e R\eta]$ are given in the last column of Tables 9–11.

All the allowed unhindered β-transitions are shown in Table 9, which in this

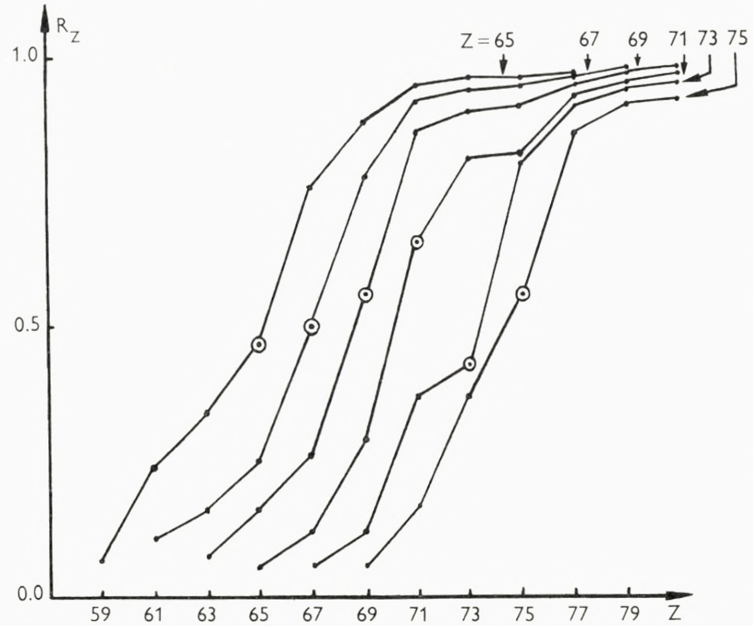

Fig. 6 a.

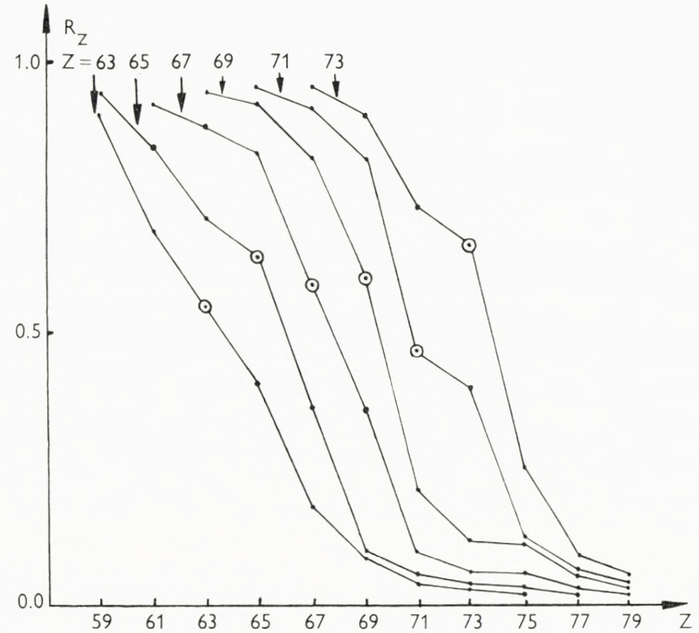

Fig. 6 b.

case happens to include only transitions of the first group. Therefore, the values of $\log(ft)_e$ are close to each other and the inclusion of superfluidity effects does not noticeably alter them.

In Table 10 the allowed hindered β-transitions are shown. In this case there are β-transitions referred to both the first and second groups and therefore the inclusion of the superfluid corrections are of greater importance (see especially the transitions between states $532\uparrow$ and $521\uparrow$). However, in these cases, the average field changes noticeably in passing from one nucleus to another. The transitions between the states $[404]\downarrow$ and $[624]\uparrow$ show unexplained fluctuations. A possible cause may be that hindered β-transitions are more sensitive to changes in the average field compared to unhindered ones.

We present in Table 11 the first forbidden unhindered β-transitions. Improved agreement is obtained between the calculated and experimental values of $\log ft$ for most of the given transitions. However, the superfluid corrections for the transition between states $[411]\uparrow$ and $[521]\uparrow$ increase the difference between the probabilities of the first and the second transitions. For the transitions between states

$$\zeta_z = \{1/2 + [411]\} \rightleftarrows \zeta_n = \{1/2 - [510]\},$$
$$\zeta_z = \{7/2 + [404]\} \rightleftarrows \zeta_n = \{7/2 - [503]\} \text{ and}$$
$$\zeta_z = \{7/2 + [404]\} \rightleftarrows \zeta_n = \{9/2 - [505]\},$$

which refer to the second group, the superfluid corrections are very important.

The β-transition probabilities observed in the strongly deformed odd-A nuclei are summarized in [16]. They are classified in the following manner:

$$4.5 < \log (ft)_e < 5.0 \qquad au$$
$$6.0 < \log (ft)_e < 7.5 \qquad ah$$
$$5.5 < \log (ft)_e < 7.5 \qquad 1\,u$$
$$7.5 < \log (ft)_e < 8.5 \qquad 1\,h.$$

In Tables 9—11 we write down $\log (ft)_e$ and $\log [(ft)_e R\eta]$ for the β-transitions in odd-A nuclei. The classification of the β-transitions in the strongly deformed odd-A nuclei is approximately represented as:

$$4.0 < \log [(ft)_e R\eta] < 4.7 \qquad au$$
$$5.5 < \log [(ft)_e R\eta] < 6.5 \qquad ah$$
$$5.5 < \log [(ft)_e R\eta] < 6.5 \qquad 1\,u.$$

In passing from the $\log (ft)_e$ to the $\log [(ft)_e R\eta]$ classification, there appears to be a tendency for a narrowing of the intervals. It may be noted that, for the $1\,u$ β-decays, the transitions of the type $[402]\uparrow \longleftrightarrow [512]\downarrow$ appear to have $\log (ft)$ values somewhat larger than the rest of the $1\,u$ transitions.

The comparison of the given β-transition probabilities between identical states in the odd and even nuclei can only be made when the change of the spin ΔI in the decay is the same and when there are no additional selection rules of the type of K and Λ forbiddenness for the transitions in the even nuclei. Only in these cases can one calculate $\log ft$ for β-decays in the even nuclei from experimental data on the odd-A nuclei.

The detailed analysis of the β-decay probabilities and the calculations of the values of $\log (ft)_r$ are made in reference[24].

6. Conclusion

The general properties of the superfluid model of strongly deformed nuclei are investigated in this paper. It is shown that, firstly, the specific features of the superfluid model are very essential in considering the properties of these nuclei and, secondly, that the mathematical method of the model is self-consistent and convenient for quantitative investigations.

In the investigation of the β-decay probabilities and of the properties of two-quasiparticle excited states of the even-even nuclei, our calculations are based on the experimental date of the single-particle levels of the odd-A nuclei and the pairing energies. Thus, our calculations are independent of the shortcomings of the unified model, the calculations themselves are quite unambiguous, and no new parameters are introduced.

In the analysis of the β-decay probabilities it was found useful to classify the transitions into three groups; the superfluid corrections are of special importance for transitions belonging to group II.

The comparison of the experimental data with the calculated energy levels of the even-even nuclei and the β-decay probabilities, made in ref. 24, shows rather good agreement between theory and experiment, and indicates that the two-quasiparticle aspect of the superfluid model can be a good basis for the analysis of the properties of the even-even nuclei.

Acknowledgements

The author expresses his deep gratitude to Professor N. N. Bogolubov for his constant interest in this investigation and for fruitful discussions, to N. N. Govorun, N. I. Pyatov and I. N. Silin for writing the program and performing the numerical calculations.

Part of this work has been carried out during the author's stay at the Institute for Theoretical Physics, University of Copenhagen, while he was on leave from the Joint Institute for Nuclear Research, Dubna, USSR. He is grateful to Professor A. Bohr for many discussions and valuable remarks. Thanks are also due Dr. C. J. Gallagher and Professor B. R. Mottelson for fruitful discussions, and Dr. B. Easlea for his assistance with some of the English language.

The author would also like to take this opportunity to thank Professor Niels Bohr and the members of the Institute for their hospitality.

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Indleveret til Selskabet den 5. juli 1961.
Færdig fra trykkeriet den 30. december 1961.

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